

AD-A055 724

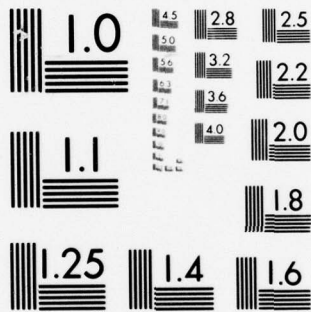
GEORGE WASHINGTON UNIV/ WASHINGTON D C INST FOR MANAG--ETC F/G 14/4
ANALYZING AVAILABILITY AND READINESS USING TRANSFER FUNCTION MO--ETC(U)
MAR 78 N D SINGPURWALLA
SERIAL-T-369

N00014-75-C-0729
NL

UNCLASSIFIED

| OF |
AD
A055724





MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

FOR FURTHER IRAN

12 R

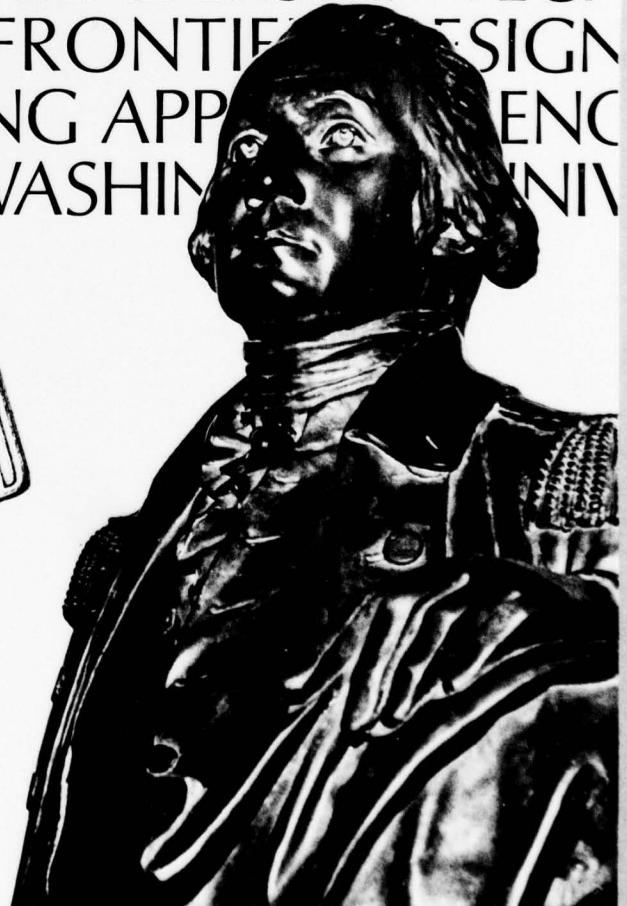
AD A 055724

AD No. _____
DDC FILE COPY

THE
GEORGE
WASHINGTON
UNIVERSITY

78 06 23 069

STUDENTS FACULTY STUDY R
ESEARCH DEVELOPMENT FUT
URE CAREER CREATIVITY CC
MMUNITY LEADERSHIP TECH
NOLOGY FRONTIER DESIGN
ENGINEERING APP ENO
GEORGE WASHINGTON UNIV



INSTITUTE FOR MANAGEMENT
SCIENCE AND ENGINEERING
SCHOOL OF ENGINEERING
AND APPLIED SCIENCE

AD No. _____
DDC FILE COPY

AD A 055724

ANALYZING AVAILABILITY AND READINESS USING TRANSFER
FUNCTION MODELS AND CROSS SPECTRAL ANALYSIS.

by

Nozer D. Singpurwalla

Scientific rept.

29 Mar 78

Serial-T-369
29 March 1978

32 p.

N00014-75-C-0729
AT(49-24)-0211

The George Washington University
School of Engineering and Applied Science
Institute for Management Science and Engineering

Jointly Sponsored by
Contract AT(49-24)-0211
Nuclear Regulatory Commission

and

Program in Logistics
Contract N00014-75-C-0729
Project NR 347 020
Office of Naval Research

DDC
RECEIVED
JUN 26 1978
F

This document has been approved for public
sale and release; its distribution is unlimited.

406 743 78 06 23 069

506

NONE

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER T-369	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) ANALYZING AVAILABILITY AND READINESS USING TRANSFER FUNCTION MODELS AND CROSS SPECTRAL ANALYSIS		5. TYPE OF REPORT & PERIOD COVERED SCIENTIFIC
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) NOZER D. SINGPURWALLA		8. CONTRACT OR GRANT NUMBER(s) N00014-75-C-0729 NRC AT(49-24)-0211
9. PERFORMING ORGANIZATION NAME AND ADDRESS THE GEORGE WASHINGTON UNIVERSITY PROGRAM IN LOGISTICS WASHINGTON, D. C. 20037		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS OFFICE OF NAVAL RESEARCH CODE 430 D ARLINGTON, VA. 22217		12. REPORT DATE 29 March 1978
		13. NUMBER OF PAGES 29
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) NONE
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) TRANSFER FUNCTION MODELS CROSS SPECTRAL ANALYSIS NUCLEAR POWER PLANT DATA ANALYSIS TIME SERIES ANALYSIS INTERRELATIONSHIPS BETWEEN OPERATING TIMES AND DOWN TIMES		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) If we look at the literature of reliability and life testing we do not see much on the use of the powerful methods of time series analysis. In this paper we show how the methods of multivariate time series analysis can be used in a novel way to investigate the interrelationships between a series of operating (running) times and a series of maintenance (down) times of a complex system. Specifically, we apply the techniques of cross spectral analysis to help us obtain a Box-Jenkins type transfer function model for		

DD FORM 1 JAN 73 1473

EDITION OF 1 NOV 65 IS OBSOLETE
S/N 0102-014-6601

NONE

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

next
Page

NONE

SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

No. 20 Abstract continued

the running times and the down times of a nuclear reactor. A knowledge of the interrelationships between the running times and the down times is useful for an evaluation of maintenance policies, for replacement policy decisions, and for evaluating the availability and the readiness of complex systems.

ACCESSION #	on	<input checked="" type="checkbox"/>
NTIS	on	<input type="checkbox"/>
DDC	on	<input type="checkbox"/>
UNANNOUNCED		
JUSTIFICATION		
BY	DISTRIBUTION/AVAILABILITY CODES	
		SPECIAL
D.		
A		

NONE

SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

THE GEORGE WASHINGTON UNIVERSITY
School of Engineering and Applied Science
Institute for Management Science and Engineering

Abstract
of
Serial T-369
29 March 1978

ANALYZING AVAILABILITY AND READINESS USING TRANSFER
FUNCTION MODELS AND CROSS SPECTRAL ANALYSIS

by

Nozer D. Singpurwalla

If we look at the literature of reliability and life testing we do not see much on the use of the powerful methods of time series analysis. In this paper we show how the methods of multivariate time series analysis can be used in a novel way to investigate the interrelationships between a series of operating (running) times and a series of maintenance (down) times of a complex system. Specifically, we apply the techniques of cross spectral analysis to help us obtain a Box-Jenkins type transfer function model for the running times and the down times of a nuclear reactor. A knowledge of the interrelationships between the running times and the down times is useful for an evaluation of maintenance policies, for replacement policy decisions, and for evaluating the availability and the readiness of complex systems.

Research Jointly Sponsored by
Nuclear Regulatory Commission
and
Office of Naval Research

THE GEORGE WASHINGTON UNIVERSITY
School of Engineering and Applied Science
Institute for Management Science and Engineering

ANALYZING AVAILABILITY AND READINESS USING TRANSFER
FUNCTION MODELS AND CROSS SPECTRAL ANALYSIS

by

Nozer D. Singpurwalla

1. Introduction and Summary

The investigation reported here was undertaken to determine if a stochastic interrelationship exists between the running times and the down times of the Robinson Nuclear Power Plant. The data was provided to us by the Probabilistic Analysis Staff of the Nuclear Regulatory Commission (NRC). Our goal was to understand what the data were telling us about the relationship between the series of running times and the series of down times. One way of achieving this goal is to obtain a Box-Jenkins (1976) type of "transfer function model" between the running times and the down times. The interpretation and uses of a transfer function model for the situation considered here are discussed in Section 1.1.

A first step in the analysis of the data was its careful screening. This was done in order to eliminate those observations that were judged to be questionable or that had arisen under unusual circumstances. Such observations introduce spurious autocorrelations and cross correlations, and thus tend to obscure the identification of a simple relationship that may exist between the running times and the down times.

We would like to emphasize that, for an analysis of data of the type discussed here (often referred to as "messy data"), an examination and

screening of the data prior to model building are important preliminary operations. If one neglects to perform these operations, one may face the frustrating task of attempting to fit several transfer function models, none of which may be satisfactory.

In Figures 1.1 and 1.2, we display a time sequence plot of the screened down times X_t and the corresponding screened running times Y_t , $t=1,2,\dots,28$. Note that X_1 represents the first down time, X_2 the second down time, and so on, whereas Y_1 denotes the first running time, Y_2 the second running time, and so on. Note also that the two plots are not drawn to the same scale. In Figure 1.3 we indicate the relative positions of the X_t 's and the Y_t 's, $t=1,2,\dots$.

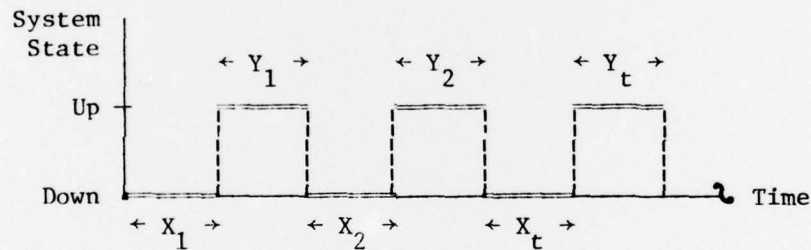


Figure 1.3 State of the system versus time.

In transfer function model building observations must be considered in pairs. In our case, the paired observations will be

$$(X_1, Y_1), (X_2, Y_2), \dots, (X_{28}, Y_{28}) .$$

In considering the above pairs, we will have to bear in mind that X_t precedes Y_t in chronological time.

In Table 1.1 we present the actual values of the screened down times X_t and the corresponding screened running times Y_t , $t=1,2,\dots,28$.

If changes in a series of observations Y_t , $t=1,2,\dots$, tend to be anticipated by changes in another series of observations, say X_t , $t=1,2,\dots$,

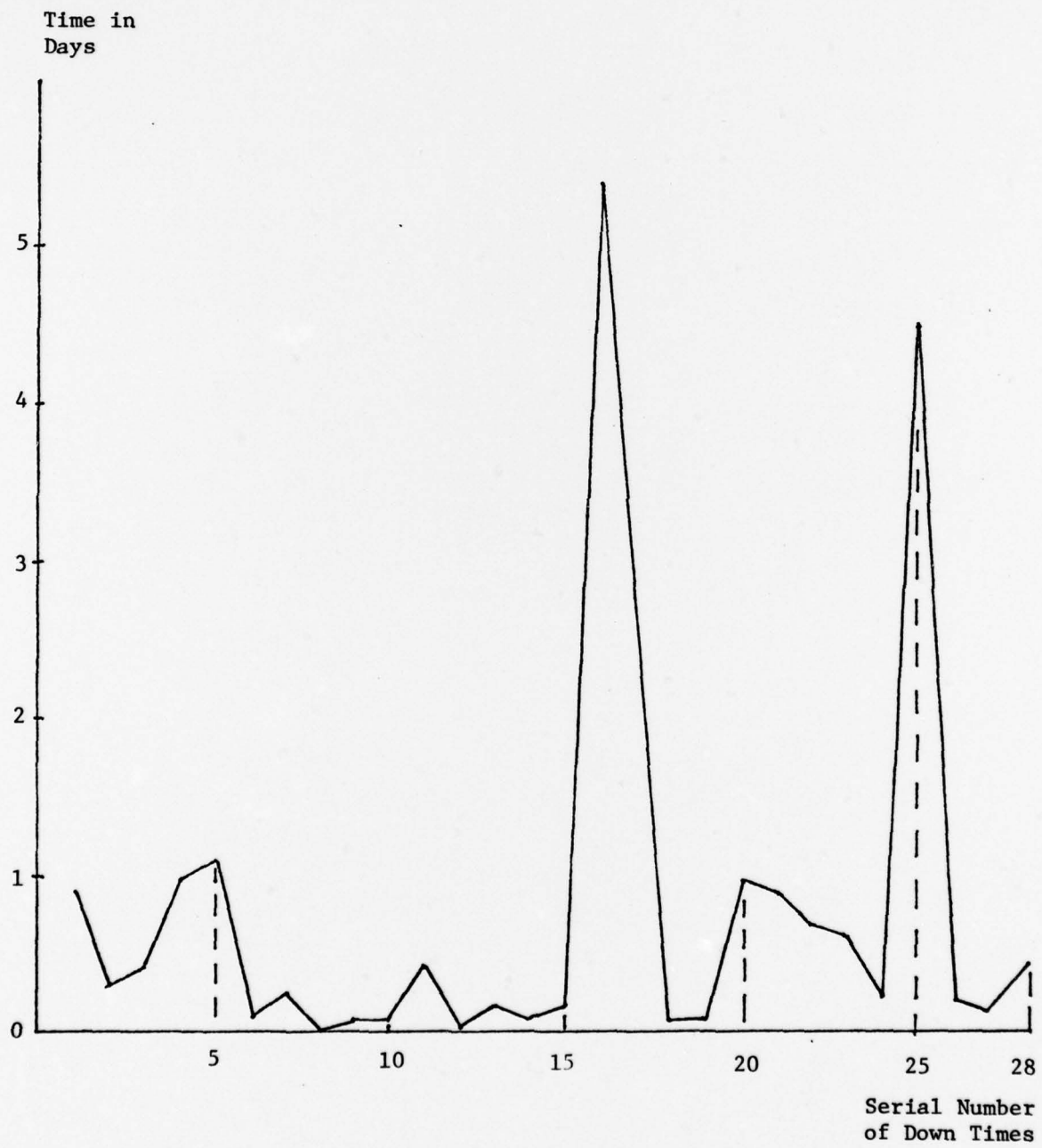


Figure 1.1 Time series plot of screened down times.

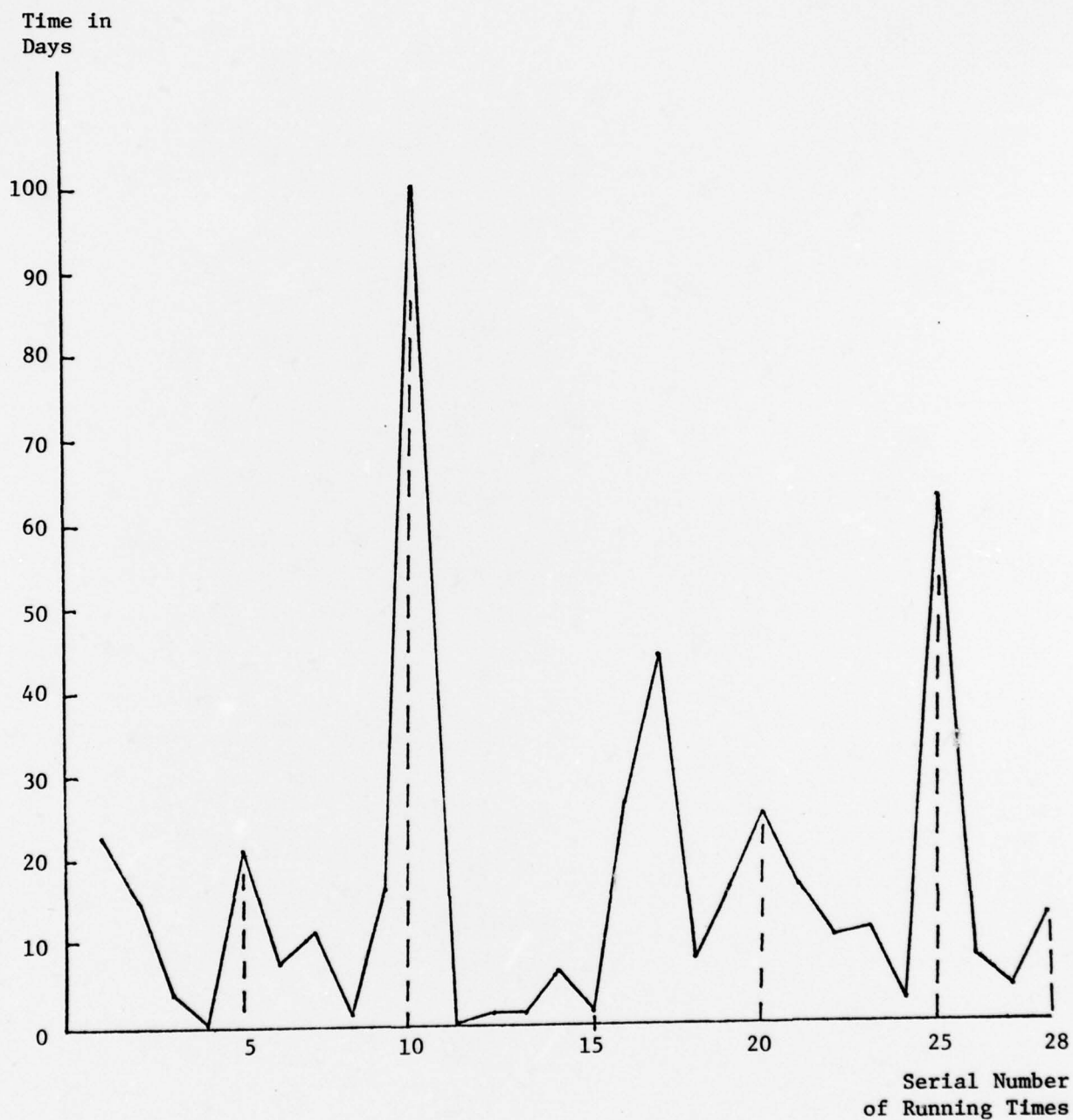


Figure 1.2 Time series plot of screened running times.

TABLE 1.1
SCREENED DOWN TIMES AND RUNNING TIMES
OF THE ROBINSON NUCLEAR POWER PLANT

Down Times (days): x_t	Prewhitened Down Times (days): α_t	Running Times (days): y_t	Prewhitened Running Times (days): β_t
0.87	0.07	22.83	6.40
0.26	-0.54	15.75	-1.43
0.42	-0.30	3.50	-11.55
0.95	-0.09	0.00	-14.50
1.13	0.19	21.00	3.88
0.06	-0.78	8.00	-13.17
0.19	-0.47	11.00	-5.09
0.02	-0.84	1.00	-16.82
0.08	-0.81	16.00	-1.18
0.06	-0.86	101.00	80.84
0.40	-0.55	0.58	27.76
0.01	-0.99	1.00	2.97
0.15	-0.75	0.83	-10.46
0.08	-0.89	6.90	-11.44
0.17	-0.80	0.83	15.85
5.48	4.51	27.00	8.86
2.81	1.21	44.00	22.01
0.06	-0.13	8.00	11.43
0.08	0.13	17.00	7.25
0.98	0.30	25.00	8.48
0.87	0.04	17.00	-0.57
0.70	-0.03	10.30	-3.64
0.63	-0.11	11.36	-3.64
0.19	-0.59	2.81	-13.93
4.60	3.85	63.00	47.04
0.19	-1.19	8.00	-17.05
0.14	0.13	3.90	-3.48
0.41	-0.12	12.50	-0.50

then X_t is said to be a *leading indicator* of Y_t . In our case it is reasonable to assume that the down times X_t are a leading indicator of the running times Y_t . Based upon this consideration, and together with an analysis of the available data, albeit *an insufficient amount*, the best transfer function model we have identified and fitted is given as

$$Y_t - 10.630 = 6.6X_t - 0.55X_{t-1} . \quad (1.1)$$

1.1 Interpretation and uses of the transfer function model

Transfer function models are generally used to forecast the future values of a time series Y_t (in our case the running times) given the previous values of the leading indicator series X_t (the down times in our case). However, forecasts of the running times based upon the previous and present values of the down times, via Equation (1.1), must be used with great caution for the following two reasons. First, the occurrence of unforeseen but rare circumstances may cause the future running times to be abnormally small (or even large). An example of this is a reactor shut-down due to an unforeseen operator error. Second, since Equation (1.1) is based on only 28 observations, it may not be too reliable as a model for forecasting. We can, however, make several observations of practical interest based upon Equation (1.1).

We first note that previous values of Y_t , such as Y_{t-1}, Y_{t-2}, \dots , etc., do not appear in Equation (1.1). This implies that the running time history gives us little information about the *individual* future running times; that is, the next running time may be unpredictable from a knowledge of the previous running times. However, future running times *on the average* may be estimated from previous running times.

An important consequence of Equation (1.1) is that the running times Y_t appear to be strongly influenced by the immediately preceding down times X_t . Since the down times generally correspond to maintenance actions, we can make the following conjecture:

Conjecture: Barring unforeseeable circumstances, and confining ourselves to the limits of the observed data, the operating times are, on the average, increased by a factor of about six per unit increase in the maintenance times.

An explanation to support the above conjecture is that the more thorough a job of repair that is performed, the longer the next running time becomes. This is perhaps one of the most important conclusions that can be reached from our analyses.

Since the coefficient of -0.55 associated with X_{t-1} is small compared to the coefficient of 6.6 associated with X_t , we will ignore the effect of X_{t-1} on Y_t . Even though the transfer function model is obtained after an involved analysis, as discussed in the remainder of this report, the simplicity of Equation (1.1) suggests that a plot of Y_t versus X_t , $t=1,2,\dots,28$, should be approximately linear. The actual plot confirms the reasonableness of Equation (1.1), including the values of its coefficients.

In conclusion, for the situation considered here the transfer function model is more effective as a tool that gives us some insight into the manner in which the system operates, rather than as a tool that can give us reliable forecasts of future running times.

The remainder of this paper is devoted to a discussion of the pertinent details that lead us to our model. In Section 2, by way of presenting some aspects of transfer function model building, we also introduce some terminology and notation. In Section 3, we present an analysis of our data.

In what follows, we require the reader to have some familiarity with the material in Box and Jenkins (1976) and with that in Jenkins and Watts (1968).

2. Transfer Function Models and Their Estimation

Univariate transfer function models as described by Box and Jenkins (1976) are models that specify the stochastic interrelationships between two time series. They are more general than regression models with lag structures on predetermined variables, in that the dependent variable can also have a lag structure. In addition, the transfer function models can have a superimposed error structure which may be of a very general nature. Engineers often refer to error with the term "noise," and "white noise" refers to errors that are independent and identically distributed.

There are two equivalent representations of a univariate transfer function model. One is the infinite or reduced form, and the other is the finite form. In the infinite form, the output series, say Y_t (in our case the running time), is explicitly represented as a function of the input series X_t (in our case the down time) and its lagged (previous) values; that is,

$$Y_t = v_0 + v_1 X_{t-1} + v_2 X_{t-2} + \dots + N_t, \quad (2.1)$$

where the constants v_0, v_1, \dots are called the *impulse response weights*.

In cases where there is no immediate response, one or more of the initial v 's, say v_0, v_1, \dots, v_{b-1} , is equal to zero. The process N_t represents noise, which is assumed to be independent of the level of the input series, but is additive with respect to the influence of the input; N_t can have any general structure.

It can be shown that an equivalent representation of the model given by Equation (2.1) is the following finite form:

$$Y_t - \delta_1 Y_{t-1} - \dots - \delta_r Y_{t-r} = \omega_0 X_{t-b} - \omega_1 X_{t-b-1} - \dots - \omega_s X_{t-b-s} + N_t, \quad (2.2)$$

where the δ 's, the ω 's, and b are unknown constants. The constant b associated with the leading indicator series X_t indicates which of the previous values of X_t affect the present Y_t . In our application, the

value of b represents the number of previous maintenance times affecting the present running time.

A first step towards estimating the transfer function model is a tentative identification of the values of r , b , and s . This can be accomplished by an examination of the estimated impulse response weights \hat{v}_k , $k=0,1,2,\dots$. A plot of \hat{v}_k versus k is known as the *impulse response function*.

There are two general approaches for obtaining the impulse response function. The first one, outlined by Box and Jenkins (1976, p. 379), is based on a "prewhitening" of the input series. Prewhitening the input series means fitting a time series model to the X_t series such that the residuals from the model, say α_t , are independent and identically distributed random variables with mean zero and a constant variance. When the \hat{v}_k are estimated using the prewhitening of the input series procedure, their neighboring values tend to be correlated. Thus the graph of the impulse response function tends to be misleading. This ultimately affects our ability to obtain a realistic transfer function model. We are therefore interested in considering an alternate approach for estimating the impulse response weights.

The second approach for estimating the impulse response weights involves the use of "cross spectral analysis." Such an approach removes the difficulties associated with the problem of the correlated estimates of v_k , and also provides us with some additional insight into the nature of the dependencies between the input and the output series. These are illustrated at the end of Section 3.

Once the impulse response function is obtained, we can isolate the noise series, N_t , by using Equation (2.1). Specifically, we estimate the noise series by

$$\hat{N}_t = Y_t - \hat{v}_0 - \hat{v}_1 X_{t-1} - \hat{v}_2 X_{t-2} - \dots \quad (2.3)$$

A knowledge of \hat{N}_t , plus a knowledge of the tentative values of r , b , and s , helps us to estimate the parameters of the transfer function model, Equation (2.2). One way of accomplishing this is by using the TIMES program package described by Willie (1977).

The adequacy of the proposed model can be checked by an analysis of the residues from the model. The details of such an analysis are given in Box and Jenkins (1976, p. 392).

3. Analysis of Running Time and Down Time Data

Data on the operating history of nuclear reactors are generally available showing dates on which the reactors ceased operation and the duration of the stoppage. Among other facts, the reasons for the stoppage are also given. Stoppages are categorized according to whether they were scheduled or forced. In addition to this, there is a further breakdown indicating whether the stoppage was due to equipment failure, testing, refueling, regulatory reasons, operator training, administrative reasons, operational error, or other causes.

3.1 Screening the data

The data that were given to us described the Robinson Power Plant's operating history from June 1974 through April 1976. These data did not contain any stoppages due to administrative reasons, operational error, or other causes; they contained one stoppage for regulatory reasons and one stoppage for refueling. In one instance, the data contained an unrealistic combination for the cause of stoppage--a scheduled failure. In this case we used our discretion to alter it to a forced failure. Whenever there were stoppages due to operator training, these were treated as running times rather than as down times. This was done for two reasons. First, the duration of each stoppage was very short (on the average about half an hour); second, we would like to concentrate on those down times that pertain to the physical operation of the system rather than on those external to it.

The single stoppage due to a forced regulatory restriction was for a period of 3.67 hours, and since it immediately followed a forced equipment failure of 15.52 hours, it was combined with the equipment failure stoppage.

Refueling the reactor takes place annually and is generally of a very long duration. In our data, we had only one stoppage for scheduled refueling, and it was of 960 hours' duration. Since the duration of this stoppage is out of line with the duration of the other stoppages (see Table 1.1), it was excluded from consideration.

We remark here that any time a stoppage (running time), say X_t (Y_t), was excluded from consideration, its corresponding running time (stoppage) Y_t (X_t) was also excluded. This is to ensure that no bias is introduced into the relationship between the two variables of interest because of the elimination of observations of either one.

Another convention followed in our analysis arises from the fact that the raw data show the dates and the duration (in hours) of each stoppage, rather than the actual time of stoppage. We assume that each down time commenced at 0000 hours (unless in some rare instances there is a second stoppage occurring during the same day). Whenever two or more breakdowns occurred during the same day, they were combined into one down time period and the intervening operating period was ignored.

Because of the paucity of data, we chose not to distinguish between stoppages due to equipment failure and those due to testing. This is reasonable because whenever there is a forced equipment failure, maintenance and test actions on other (nonfailed) components are routine. Thus, in practice it is difficult to differentiate clearly between the consequences of equipment failure and those of testing. This strategy was suggested by some staff members of the Probabilistic Analysis Staff at NRC.

The preliminary screening and examination described above gives us a series of values of the down times X_t (in days) and a series of

values of the corresponding running times Y_t (in days). However, as we shall soon see, some further screening is necessary.

The next step in our analysis involved prewhitening the X_t series. This turned out to be quite a frustrating endeavor, since no simple univariate time series model of the Box-Jenkins type seemed to provide a reasonable fit. The difficulty turned out to have been caused by two unusually large down times due to scheduled testing and forced failure of 18.08 and 25.43 days, respectively. These were incompatible with the other down times (see Table 1.1), and thus defied the use of a simple model as a prewhitening transformation. Perhaps a time series model with an indicator variable [such as those used in the "intervention analysis" of Box and Tiao (1975)] might have been adequate for these and for the refueling stoppage, but this was not attempted. In the interest of expediency, it was preferable to eliminate the two large X_t 's and their corresponding Y_t 's. Thus, in effect, some data screening was done during the prewhitening phase. Table 1.1 presents the 28 screened values of the down times X_t and the corresponding running times Y_t . We remind the reader that the subscript t is a sequential index rather than an index representing time. That is, X_t and Y_t are not simultaneously observed in time; X_t precedes Y_t .

3.2 Transfer function model development

In Figures 3.1 and 3.2 we show plots of the autocorrelation functions of the (screened) down times X_t and the (screened) running times Y_t , respectively. Based upon these plots we are able to conclude that the two time series can be treated as stationary (see Box and Jenkins (1976), p. 174).

In Figure 3.3 we show a plot of the logarithm of the smoothed estimator of the power spectrum of the screened down times X_t , as well as

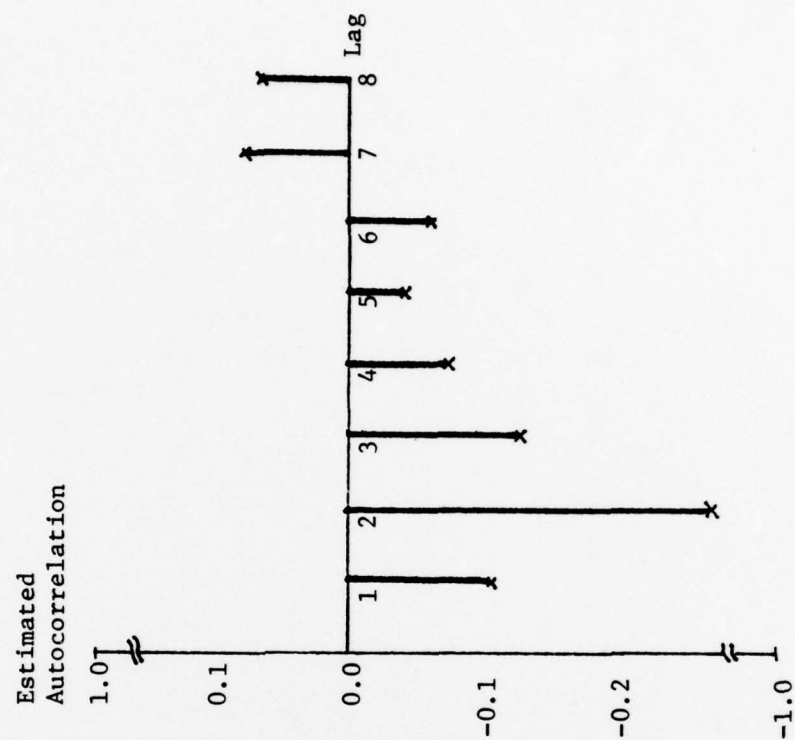


Figure 3.1 Estimated autocorrelations of down times X_t .

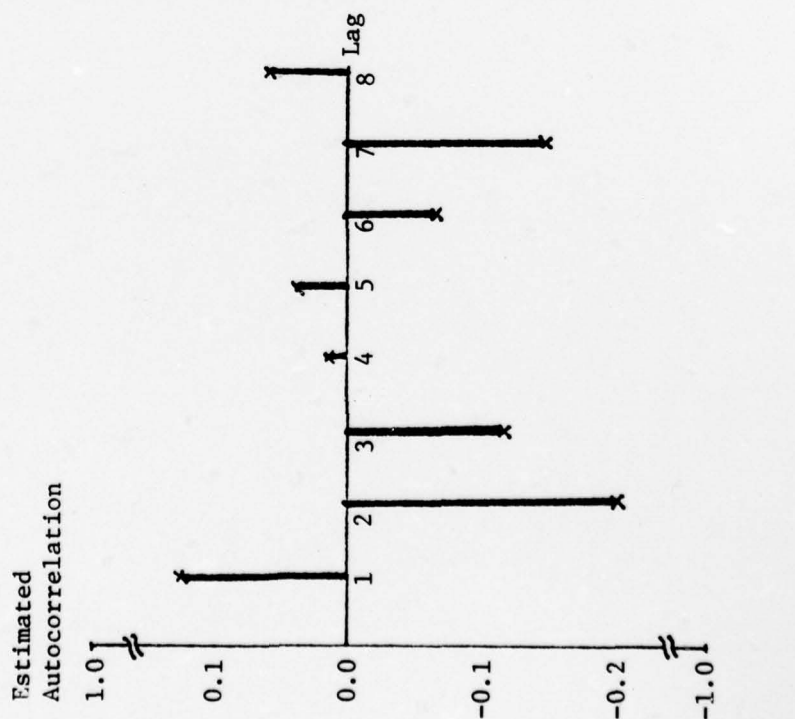


Figure 3.2 Estimated autocorrelations of running times Y_t .

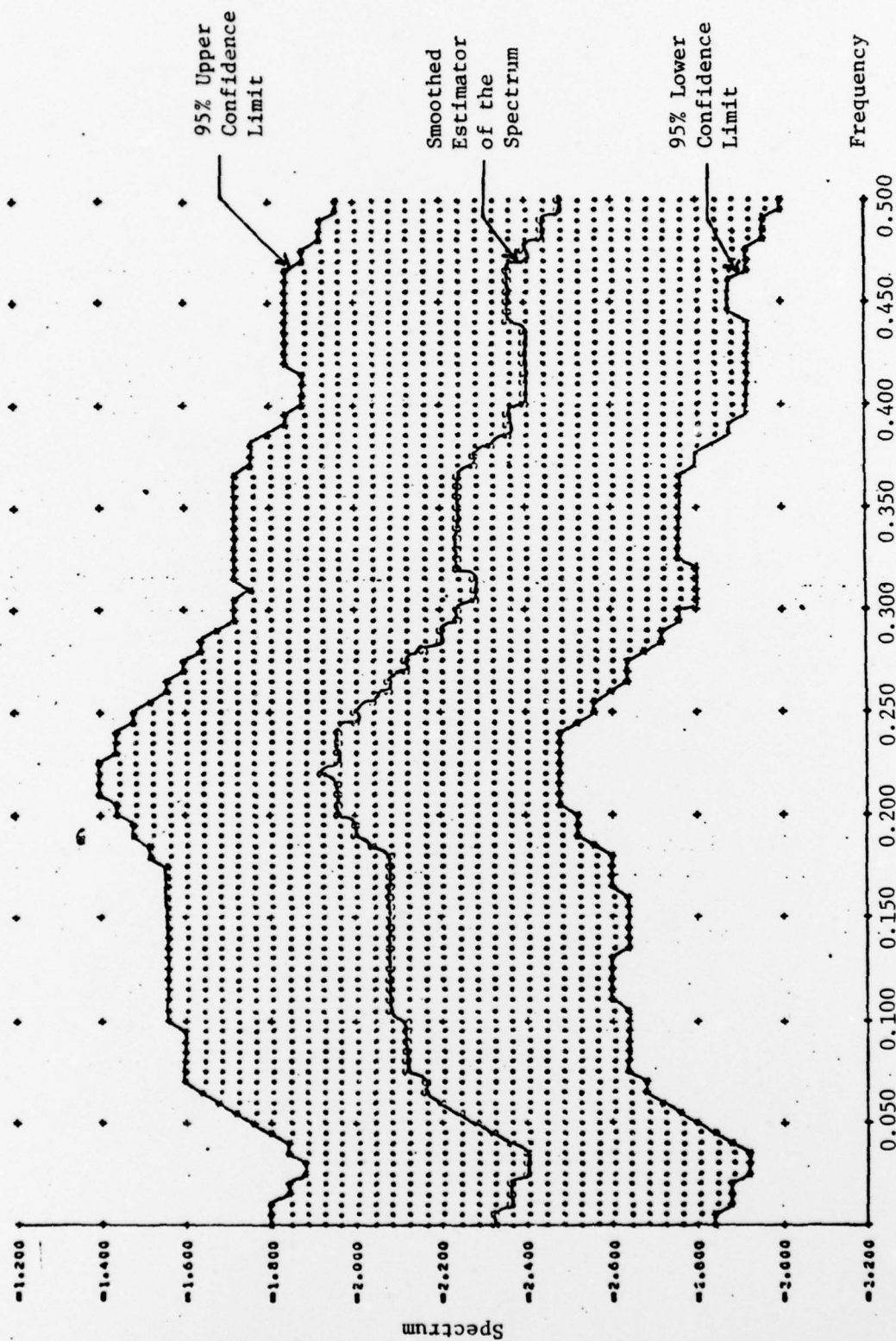


Figure 3.3 \log_{10} of the smoothed estimator of the spectrum of down times versus frequency, using rectangular window of band width .098.

the approximate 95% confidence limits. The smoothing was performed using a rectangular window of band width .098. As a matter of fact, all the smoothing that is discussed in this paper was performed using a rectangular window of band width .098. The power spectrum curve shows us how the variance of the X_t time series is distributed with frequency.

For a detailed understanding of the power spectrum, its smoothing, and the band width of a smoothing window, we refer the reader to Chapter 6 of Jenkins and Watts (1968).

Our next step involves the determination of a suitable prewhitening transformation for the down times X_t . Based upon the several models that we attempted, we conclude that a moving average process of order 3 best describes the X_t series. Specifically, we find that

$$X_t - 0.7963 = \alpha_t + 0.117\alpha_{t-1} - 0.189\alpha_{t-2} - 0.133\alpha_{t-3}, \quad (3.1)$$

where .7963 is the mean of the X_t series. The α_t represent the residuals when a moving average process of order 3 is fitted to the X_t series. If the prewhitening transformation given by Equation (3.1) is correct, then the α_t will be independently and identically distributed with a constant mean and variance.

In Table 1.1 we give the values of the α_t 's. In order to verify the appropriateness of the model given by Equation (3.1), we plot the estimated autocorrelation function and the estimated power spectrum of the α_t series. These plots are given in Figures 3.4 and 3.5, respectively. We remark that the plot of the estimated power spectrum of the α_t series given in Figure 3.5 is relatively constant as compared to the plot of the estimated power spectrum of the X_t series given in Figure 3.3. This is because the effect of prewhitening is to remove the dependencies among the X_t 's and give us a set of independent α_t . Figures 3.4 and 3.5 confirm the appropriateness of the prewhitening transformation given by Equation (3.1).

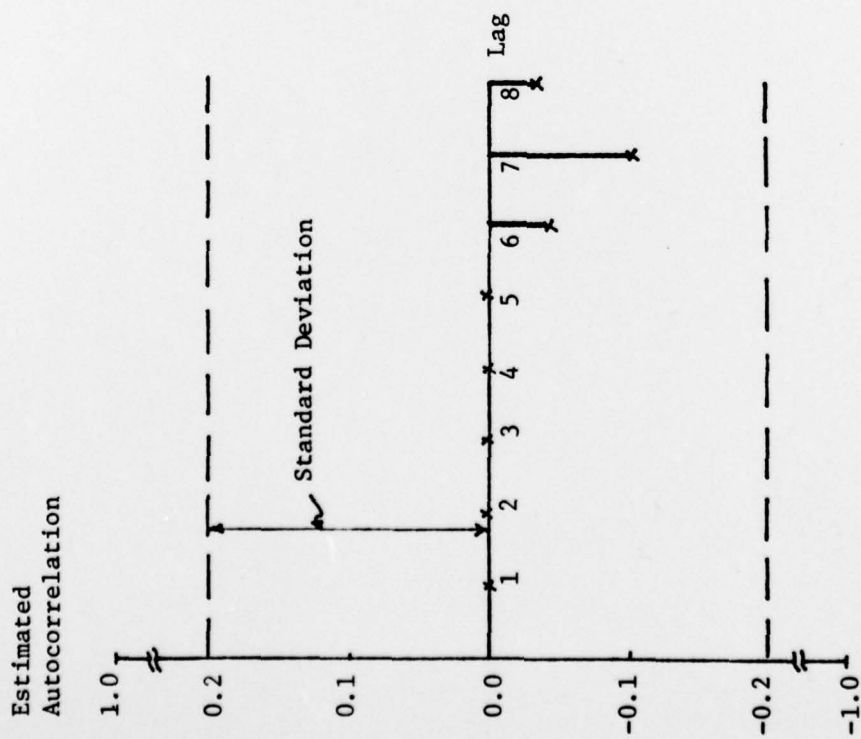


Figure 3.4 Estimated autocorrelations of residuals from the model used for prewhitening X_t .

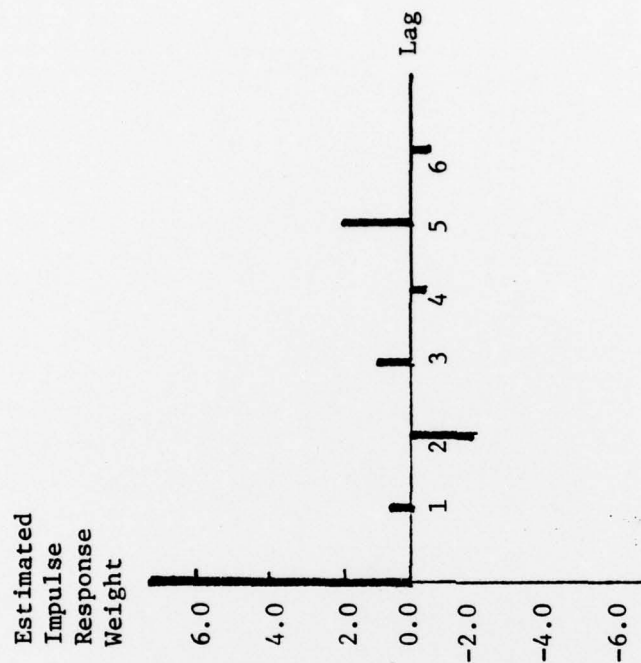


Figure 3.7 The impulse response function using the prewhitening of the input series.

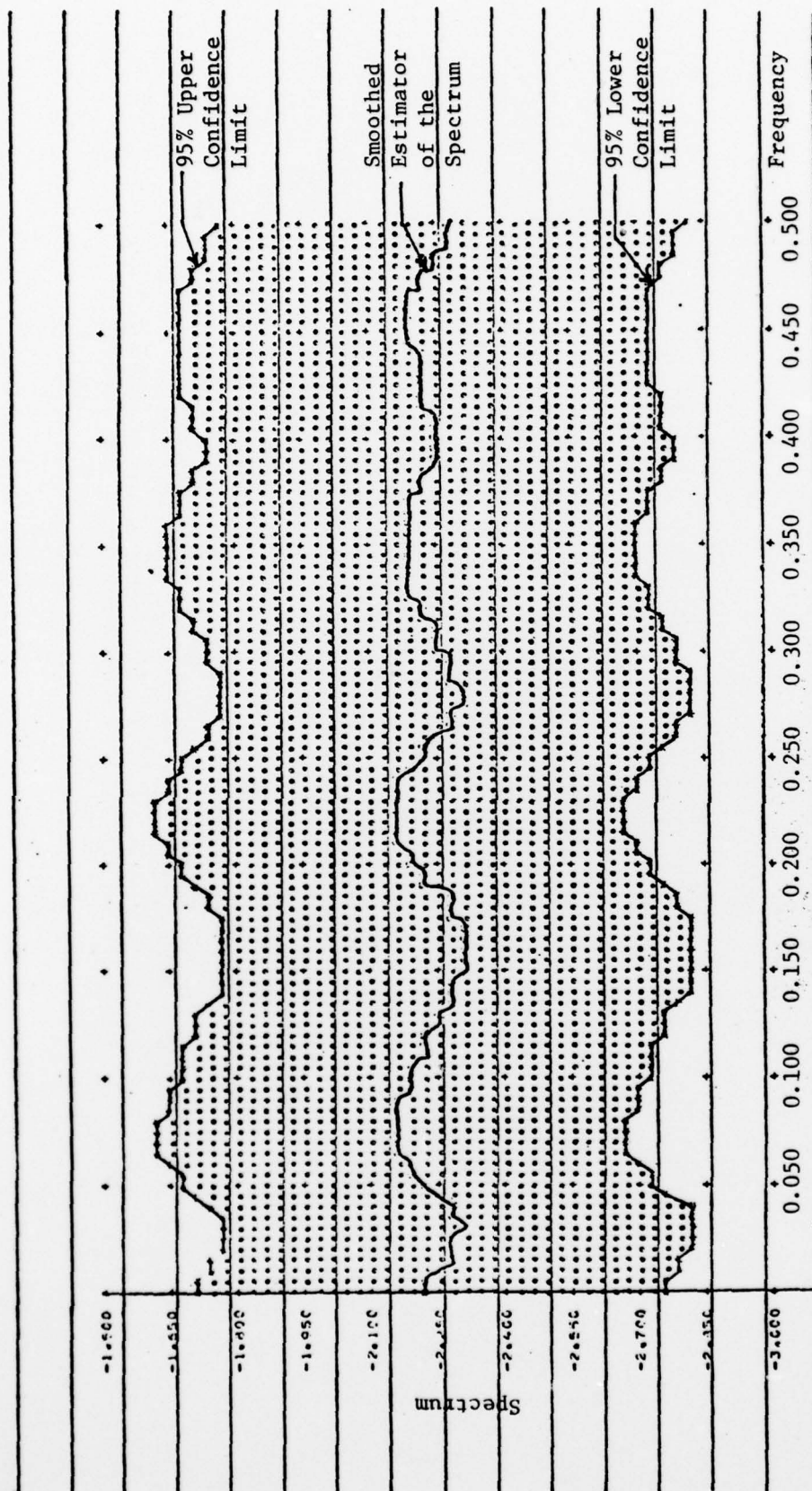


Figure 3.5 \log_{10} of the smoothed estimator of the spectrum of the prewhitened down times versus frequency, using rectangular window of band width .098.

Following Box and Jenkins (1976, p. 380), we next apply the same prewhitening transformation [i.e., the one given by Equation (3.1)] to the running times Y_t and obtain the β_t 's as residuals. In Table 1.1 we give the values of β_t under the heading "Prewhitened Running Times." In Figure 3.6 we show a plot of the smoothed estimator of the power spectrum of β_t . We remark that except at the very low frequencies, the plot of the estimated power spectrum of the β_t 's is fairly constant. Thus it appears that the prewhitening transformation given by Equation (3.1), when applied to the running times Y_t , also yields a sequence of independent and identically distributed random variables β_t .

Our next step is to obtain the cross correlation between the α_t and β_t at lags k , $k=0,1,2,\dots$. If s_α and s_β denote the estimated standard deviations of the α_t and the β_t series, respectively, and if $r_{\alpha\beta}(k)$ denotes the estimated cross correlation between the α_t and the β_t at lag k , then \hat{v}_k , an estimate of v_k , is

$$\hat{v}_k = r_{\alpha\beta}(k) \frac{s_\beta}{s_\alpha}, \quad k=0,1,2,\dots,$$

(see Box and Jenkins (1976), p. 380).

In Figure 3.7 we show a plot of the impulse response function; that is, a plot of \hat{v}_k versus k , $k=0,1,2,\dots$. We remark that in Figure 3.7, the value \hat{v}_0 is significantly larger than the other values of \hat{v}_k , $k=1,2,\dots$.

Because the neighboring values of the \hat{v}_k in Figure 3.7 tend to be correlated, we also obtained the impulse response function using the "cross spectrum" between the X_t and the Y_t series (see Jenkins and Watts (1968), p. 424). In Figure 3.8 we show a plot of the impulse

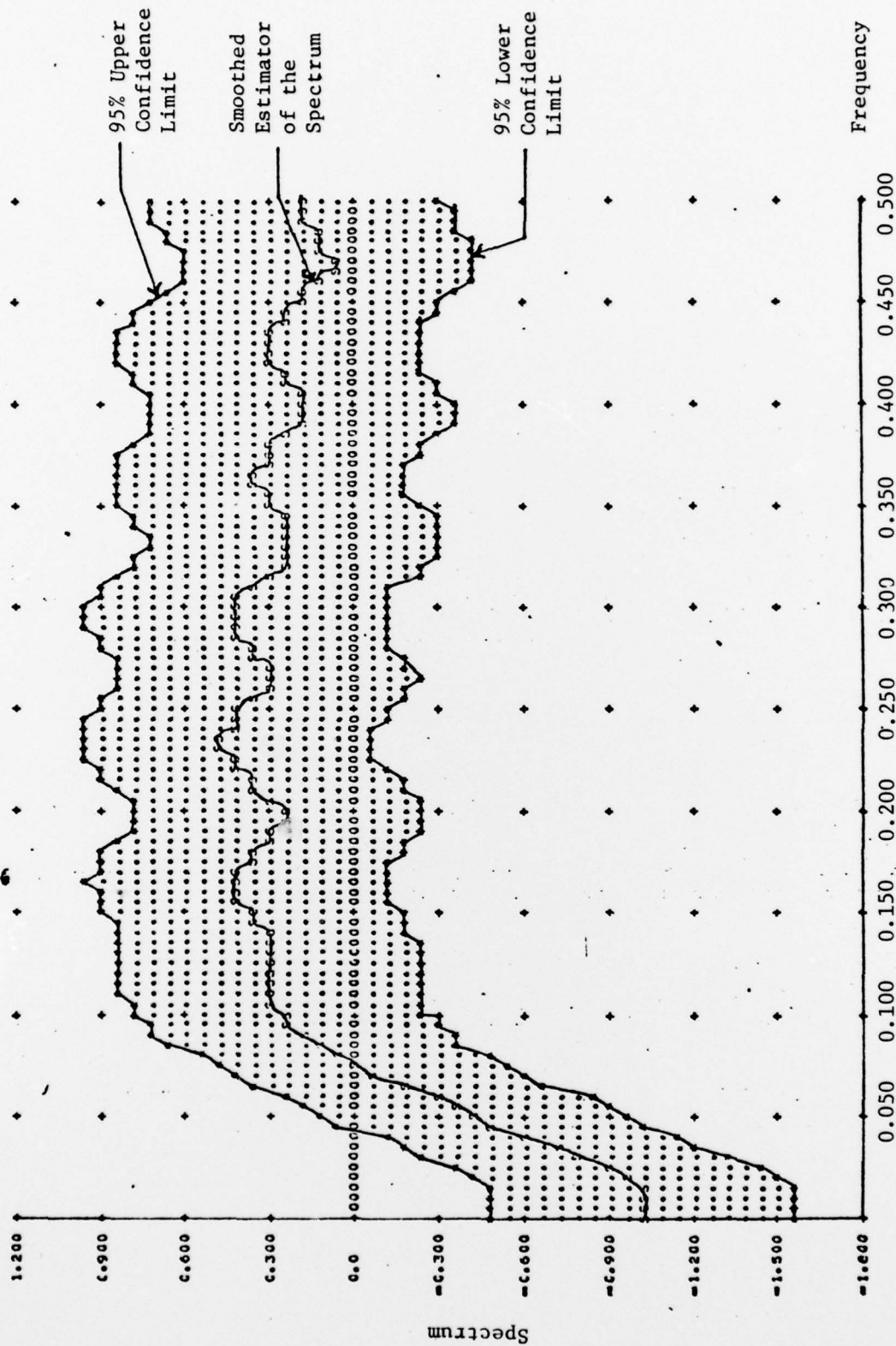


Figure 3.6 \log_{10} of the smoothed estimator of the spectrum of B_t versus frequency, using a rectangular window of band width .098.



Figure 3.8 The impulse response function using cross spectral analysis.

response function using the cross spectrum. Note that this plot is quite similar to the one given in Figure 3.7; that is, \hat{v}_0 is significantly larger than the other values of \hat{v}_k . Based upon Figures 3.7 and 3.8, we can conclude that *the greatest influence on the morning time is exerted by the down time immediately preceding it*. This is, of course, a major point of our conclusions.

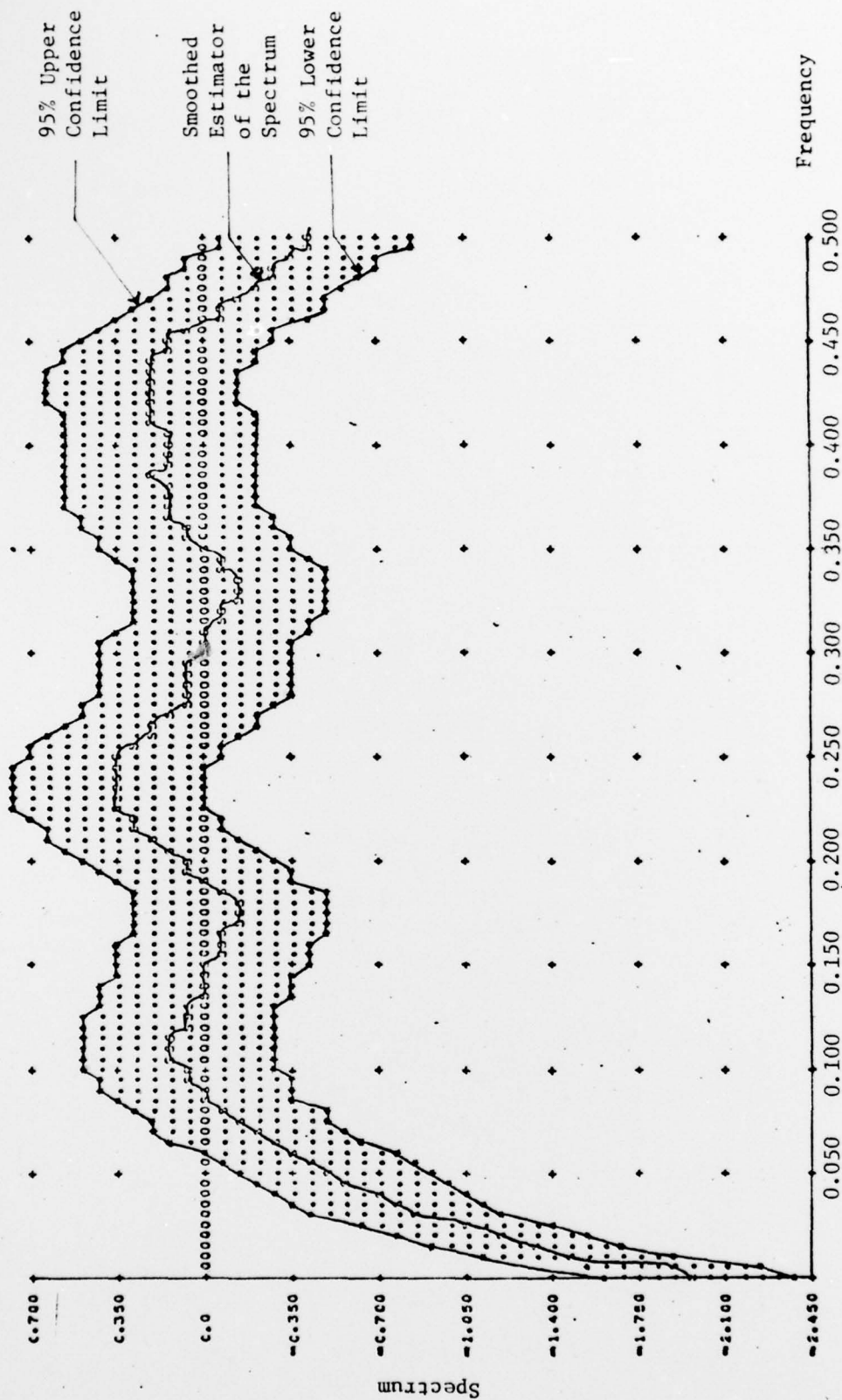
We now estimate the noise series N_t using Equation (2.3). An inspection of the estimated autocorrelation function of the estimated noise series \hat{N}_t , and a "portmanteau lack of fit test" (1.69 with 8 degrees of freedom) (see Box and Jenkins (1976), p. 290), lead us to conclude that the \hat{N}_t are independent and identically distributed. In addition to the above, we show in Figure 3.9 a plot of the smoothed estimator of the power spectrum of the \hat{N}_t . Here again, except at the very low frequencies, the estimated power spectrum of the \hat{N}_t series is fairly constant. Thus the estimated noise series \hat{N}_t can be described reasonably well by a white noise process.

A final step in the analysis involves the fitting of a transfer function model to the running times Y_t . This was accomplished by using the TIMES program package. Of the several models that were attempted, the model

$$Y_t - 10.630 = 6.6X_t - 0.55X_{t-1} \quad (3.2)$$

appears to be the best; 10.630 is the mean of the Y_t series.

In order to verify the reasonableness of the model, two diagnostic checks were suggested in Box and Jenkins (1976). One depends on the autocorrelation of the \hat{N}_t and the other depends on the estimated cross correlation between the \hat{N}_t and the α_t . For both cases a portmanteau lack of fit test was used. In the former case, the test statistic is 1.69 with 8 degrees of freedom, whereas in the latter case the test statistic is



T-369

Figure 3.9 Log_{10} of the smoothed estimator of the spectrum of the estimated noise series, using a rectangular window of band width .098.

.698 with 6 degrees of freedom. These values support the reasonableness of the proposed transfer function model.

As stated earlier, a cross spectral analysis of the X_t and the Y_t series can give us further insight into the nature of the dependencies between the two series. For example, $G(f)$, the "gain" at frequency f , behaves like the regression coefficient in a linear regression model between the output and the input at frequency f (see Jenkins and Watts (1968), p. 352). In Figure 3.10 we plot the gain of the running times on the down times at various frequencies. Another important function in cross spectral analysis is the "squared coherence" between the input and the output at frequency f . This quantity measures the correlation between the sinusoidal component of Y_t and that of X_t at frequency f . The square coherence is also in some sense a measure of the proportion of information in the Y_t series that is attributable to the X_t series. For more information on the coherence and the coherence spectrum, we refer the reader to Jenkins and Watts (1968, p. 352). In Figure 3.11 we show a plot of the coherence for the running time and down time data. Figures 3.10 and 3.11 give us some additional assurance on the dependence of the running times on the down times.

4. Summary and Conclusions

In the foregoing analysis we have demonstrated the use of time series analysis methodology for studying the interrelationships between the maintenance times and the running times of a complex system. Our analysis enables a decision maker to assess the impact of his maintenance policies on running times, or to influence the operating times by managing the maintenance times. In addition, given a down time a decision maker can, to a *limited extent*, forecast the next running time. This type of information may be very valuable, especially for large and complex systems.

Our analysis can be criticized on the grounds that it is based on an insufficient amount of data. We hope that this criticism can be overlooked in the light of the fact that our approach is to be viewed as a prototype for the analysis of reliability data involving two interrelated sources of data.

ACKNOWLEDGMENTS

The written comments by Drs. Ray Waller and Gary Tietjens, and the helpful conversations with Professor Robert Shumway and Mr. Randall Willie, are much appreciated. Mr. Mahesh Chandra's efforts on behalf of the work reported here is also acknowledged. Finally, the several comments by Professor Frank Proschan with respect to the direction and the emphasis of this report are gratefully acknowledged.

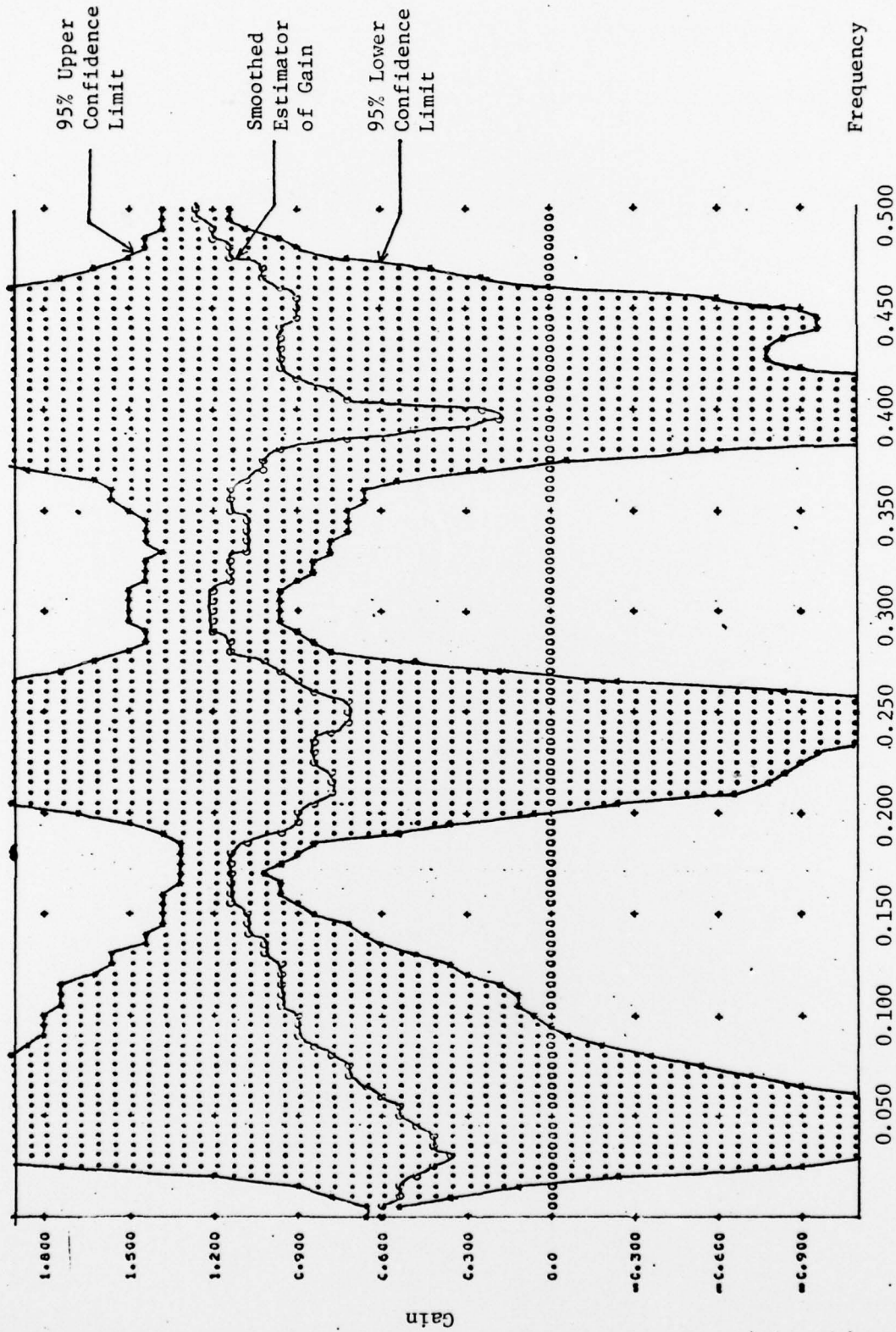


Figure 3.10 The gain versus frequency of running times on down times.

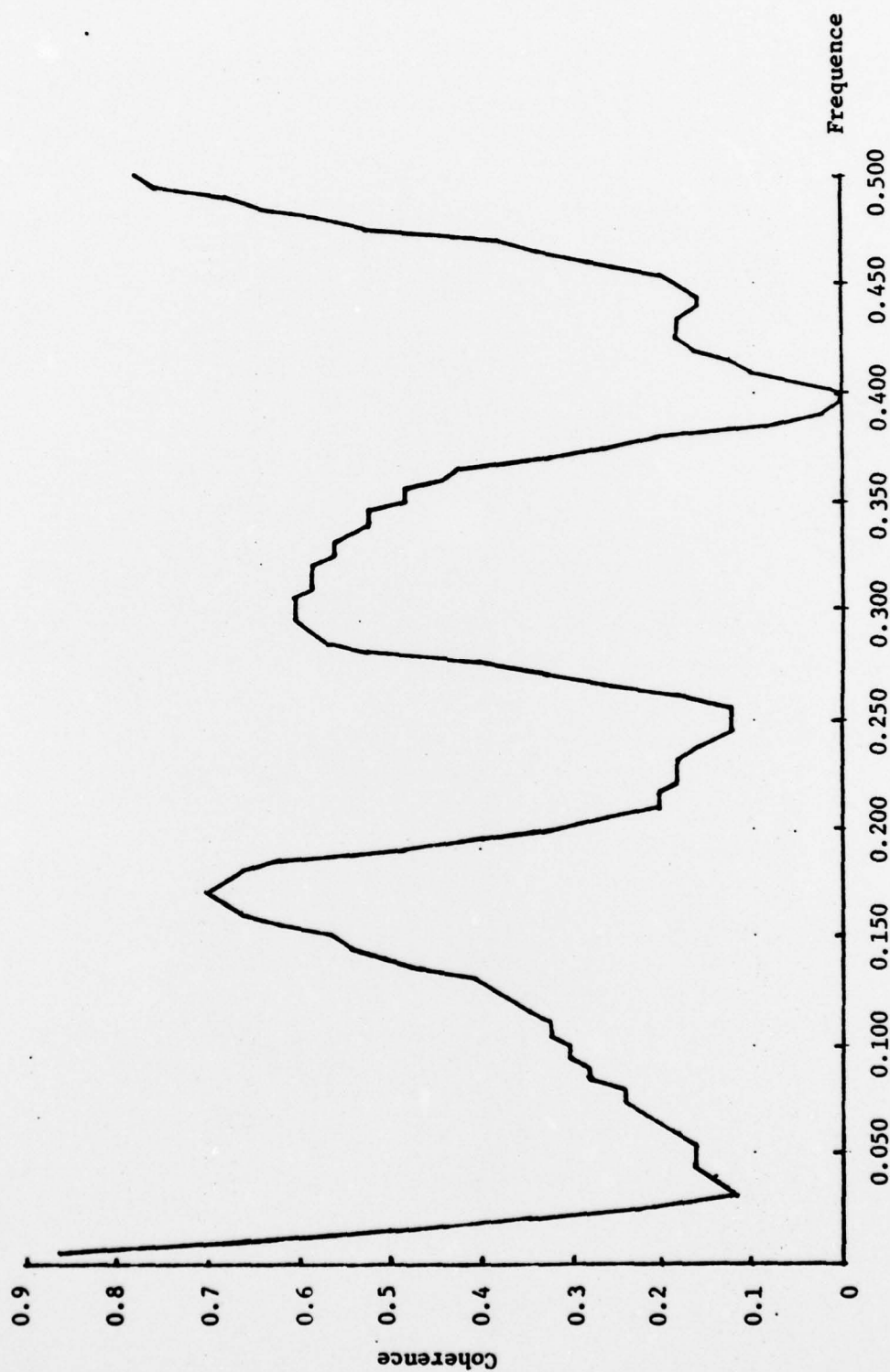


Figure 3.11 The coherence versus frequency of running times and down times.

REFERENCES

- BOX, G. E. P. and G. M. JENKINS (1976). *Time Series Analysis, Forecasting, and Control*, revised edition. Holden-Day, San Francisco.
- BOX, G. E. P. and G. C. TIAO (1975). Intervention analysis with applications to economic and environmental problems. *J. Amer. Statist. Assoc.* 70 70-79.
- JENKINS, G. M. and D. G. WATTS (1968). *Spectral Analysis and its Applications*. Holden-Day, San Francisco.
- WILLIE, R. R. (1977). Everyman's guide to TIMS. ORC 77-2, Operations Research Center, University of California at Berkeley.